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The shape of holes

Marco Bertamini[a,*], Camilla J. Croucher[b]

[a]*Department of Psychology, University of Liverpool, Eleanor Rathbone Building, Bedford Street South, Liverpool L69 7ZA, UK*

[b]*MRC Cognition and Brain Sciences Unit, University of Cambridge, Cambridge, UK*

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Abstract

The shape of holes can be recognized as accurately as the shape of objects (Palmer, S. E. (1999). *Vision science: photons to phenomenology*. Cambridge, MA: MIT Press), yet the area enclosed by a hole is a background region, and it can be demonstrated that background regions are not represented as having shape. What is therefore the shape of a hole, if any? To resolve this apparent paradox, we suggest that the shape of a hole is available indirectly from the shape of the surrounding object. We exploited the fact that observers are faster at judging the position of convex vertices than concave ones (*Perception 30* (2001) 1295), and using a figural manipulation of figure/ground we found a reversal of the relative speeds when the same contours were presented as holes instead of objects. If contours were perceived as belonging to the hole rather than the surrounding object then there would have been no qualitative difference in responses to the object and hole stimuli. We conclude that the contour bounding a hole is automatically assigned to the surrounding object, and that a change in perception of a region from object to hole always drastically changes the encoded information. We discuss the many interesting aspects of holes as a subject of study in different disciplines and predict that much insight especially about shape will continue to come from holes. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

When Ringo Starr in the animated movie Yellow Submarine picks up a round black hole and puts it in his pocket, we enjoy the joke and wit because holes have a special ontological status: they exist but they are not real objects (Casati & Varzi, 1994). From the point of view of how we perceive the shape of holes, Stephen Palmer (1999) has discussed an interesting paradox, which has its roots in the phenomenon of figure/ground organization

* Corresponding author.
E-mail address: m.bertamini@liv.ac.uk (M. Bertamini).

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described originally by the Danish Gestaltist Edgar Rubin (1921). In Palmer's words: "if the contour of the hole is assigned to the surrounding object […] how can observers then see the hole itself as having a shape?" (Palmer, 1999, p. 286). A possible solution to the

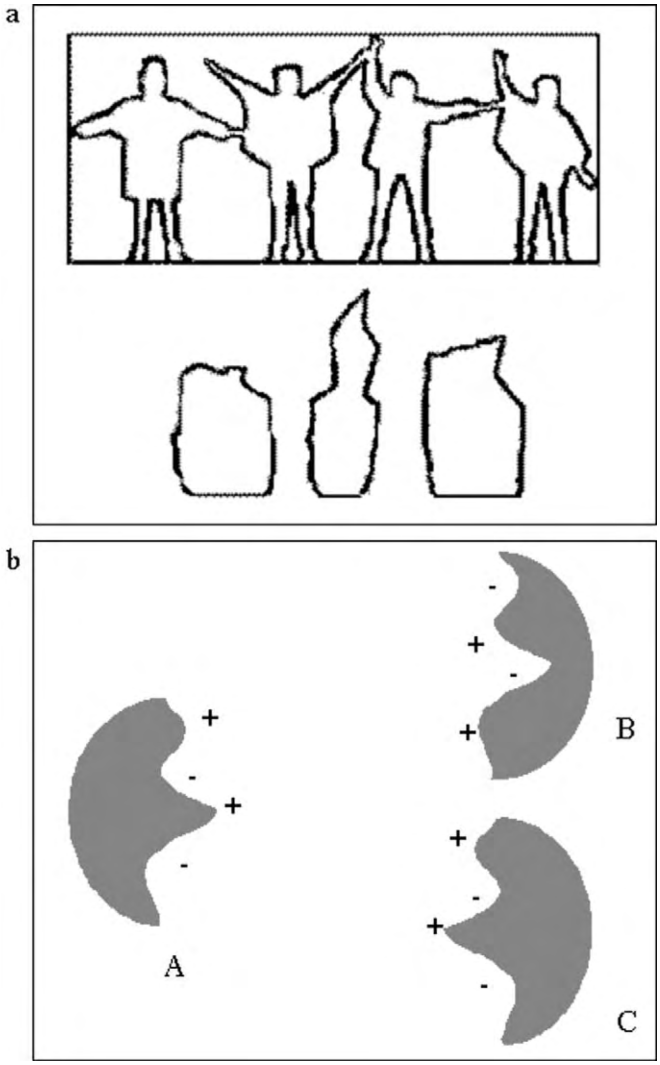

Fig. 1. (a) Because of how we organize the top scene into figures and background, the ground regions between the figures do not have shape. Notice how the regions of background isolated below are shapes that we do not perceive in the scene above (if this is parsed as a group of human semaphore signals). This demonstration is based on a similar one in Kanizsa (1979). (b) Two halves of a disk have different shapes, even though they share the same bounding contour. This demonstration is based on Attneave (1974). The positive and negative curvature of the contour has been labelled to show that when polarity changes from A to B (because of a change of where the inside of the figure is) the perceived shape is different.

paradox is that, in fact, holes do not have shape, but the surrounding object does. People may therefore judge many aspects of the hole indirectly, by looking at the object. In this paper we demonstrate this empirically by showing that the representation of a shape changes in a predicted way with a change in figure/ground organization, that is with a change from a figure to a hole.

In the introduction we will first discuss figure/ground organization and in particular the principle of unidirectional boundary belonging. There is an old Gestalt principle that says that boundaries belong to figures, and if a hole is not a figure it follows that it does not have boundaries. Next we briefly review the philosophical debate on the nature of holes, with respect in particular to the book by Casati and Varzi (1994). Casati and Varzi (1994) from a theoretical standpoint, and Palmer (1999) from a perceptual standpoint, are in agreement on the necessity to assign shape information to holes. The implication of this is that holes are qualitatively different from other background regions. We disagree with this view at least in so far as we believe in the generality of the principle that says that boundaries only belong to figures: this principle holds also in the case of holes.

1.1. Figure/ground organization and boundary belonging

When an ambiguous 2D image such as Rubin's vase is resolved into a figure and a background, the figure is thing-like, nearer the observer, closed, and its shape is clearly defined by the contour (Koffka, 1935; Rubin, 1921). The background on the other hand extends behind the figure and therefore has no shape. In other words, the contours always belong to the figure, and even when a display is ambiguous, it will never belong to both regions at the same time. In the famous vase example, the contour can belong to a vase in front of a uniform background, or to two faces in front of a uniform background. Fig. 1a is based on our favourite demonstration in Kanizsa (1979). When the scene is perceived as showing four people the space of background visible between the figures has no shape. When these regions of background are isolated as figures (as in Fig. 1a) we need to inspect the image carefully to notice that those contours were present in the scene all along.

The modern interpretation of this effect on the basis of a generic viewpoint assumption is that contours belonging to two objects are unlikely to coincide in the image. Given a cluttered world of opaque objects it is more reasonable for the visual system to interpret contours as the product of occlusion (Albert, 2000; Driver & Baylis, 1996; Hoffman, 1998). There are also theories that suggest inhibitory processes on opposite sides of a contour which would explain the fact that the ground appears shapeless (e.g. Peterson, de Gelder, Rapcsak, Gerhardstein, & Bachoud-Lévi, 2000).[1]

There is evidence to support the Gestalt principle that contours only belong to figures. In memory, people cannot recall as a shape the region that was seen as part of the background. To use again the example of the vase, people would not remember the vase if they

[1] In Peterson et al.'s model (Peterson et al., 2000; Peterson & Kim, 2001) the regions of background are processed and can affect figure/ground organization, for instance if they match memory traces. At the same time the inhibitory process may explain the fact that such regions are not consciously seen as having shape. However, the debate on how much the background regions are processed is not central to our paper. Peterson et al.'s model is in agreement with the Gestaltist position that the representation of a background region (and presumably holes) should always be weaker in a direct comparison with the representation of a figure.

saw two faces, and vice versa (Rock, 1983), and people cannot easily match contours that have changed because of a change in figure/ground assignment, so that convexities and concavities have swapped (Driver & Baylis, 1996). Moreover, in ambiguous situations, border ownership is bistable, rather than leading to cue averaging (Albert, 2001).

The problem with holes, therefore, is that their shapes should not be perceived, and therefore should not be remembered, if they behave just like any other background region (see Fig. 1a). Contrary to this prediction, Rock, Palmer, and Hume (cited in Palmer, 1999) found that people remember the shape of holes *just as well* as the shape of an object. This is what Palmer calls a paradox. He suggests that a representation for the shape of the hole must exist, maybe because when an object has a hole in it we have a representation of a hole-less object plus a representation of the hole (Palmer, 1999).

1.2. The ontology of holes

Holes can be discussed in the context of ontology (do holes have existence in themselves?), topology[2] (how would a topological change, like acquiring a hole, change an object?), and even mereology (what role do holes play in part–whole relationships?). We do believe that the most relevant description of the visual world of humans is in terms of surfaces and media (as argued by Gibson, 1979), and we also accept that in the description of shapes a crucial role is played by part structure, and in this sense mereology is relevant. However, before we can deal with the role of parts, we need to discuss some basic ontological issues.

With respect to figure/ground organization we have seen that regions of ground are not perceived as having a shape. Holes are a type of regions of background, and all regions of background (including holes) do not exist as objects. Perhaps the human visual system simply did not have any reason to develop a machinery sensitive to the shape of non-existent things.

The essay by Lewis and Lewis (1983) is an excellent exploration of the controversy on the nature of holes. On the one side we have nominalistic materialism which suggests that nothing exists but concrete material objects, and on the other side we have those who believe that entities like holes exist even though they are neither material nor objects. The Lewis and Lewis (1983) solution is to equate holes with hole-linings which are material, and claim that the rest of the problem is about language. Casati and Varzi (1994) disagree and suggest that holes are *superficial particulars*. There is a similarity between this philosophical position and Palmer's suggestion that we must be able to perceive both the objects and the holes within the objects (Palmer, 1999).

Casati and Varzi's treatment of holes is interesting for us in our study of shapes, and in agreement with us they accept the central role of surfaces (Casati & Varzi, 1994). Surfaces belong to geometry, not physics (Koenderink, 1990), and in this sense holes cannot be treated in a purely materialistic way. However, in their book Casati and Varzi also formulated a question very similar to Palmer's paradox. Taken literally, the positions of both

[2] There is a debate on the usefulness of topology to describe how the visual system analyzes shape (e.g. Chen, 2001; Hecht & Bader, 1998) but this issue is not central for our argument. In the present work we exploit holes to manipulate a type of information, namely contour curvature, which is outside the realm of topology.

Casati and Varzi and Palmer predict that the shape of a hole should be available to the observer, and they do not predict any qualitative difference between the perceived shape of a hole and the perceived shape of an object when such shapes are congruent. As we shall see, we do predict (and find) such a qualitative difference.

One last aspect of the Casati and Varzi (1994) analysis needs to be mentioned. On topological grounds they provide a classification of holes into hollows, tunnels, and cavities. The holes we are concerned with in this paper are tunnels, in that a background is always visible through our holes. Nevertheless, we would argue that our predictions apply to any holes in solid objects where the visible boundary of the hole is the projected rim of the surface as it turns inwards (self-occlusion). This will become clearer in the next section in which we set out the rationale for our predictions, but as an example one can think of the inside of a mouth. This is a hollow (if we ignore the digestive system) through which we do not see a background but rather the inside of the hole. The lips are the defining boundary of the hollow, but because the contours of the lips are concave (in the projected image) this hollow should behave like a tunnel in our analysis. We will now turn to this distinction between convex and concave contours, and an analysis based not on topology but on some simple considerations from differential geometry.

1.3. Holes and parts

It can be argued that there is an evolutionary progression from topology to affine geometry, to Euclidean geometry. Much progress has been made recently in artificial intelligence by considering the front-end visual system as a Geometer, or as a geometry engine (e.g. Koenderink, 1990; ter Haar Romeny & Florack, 2000). We do not have the space or the expertise to offer a review, but there is one specific contribution based on differential geometry which we need to introduce, because its implications will be the basis of our experiments.

Consider a world of surfaces defining solid shapes in 3D Euclidean space, and a projection onto a 2D image. At the end of the previous section we have started to use the word rim, which we need to define more clearly. A rim is the place on a surface where what is visible ends and the surface becomes invisible (due to self-occlusion). Therefore, although the rim is a specific boundary on the 3D surface, its location depends on the vantage point of the observer. Because at the rim the surface becomes invisible, the rim will project onto a 2D image as a contour. Taking a sphere as the simplest example, in parallel projection the rim corresponds to a great circle of the sphere, and the contour in the 2D image is a circle.

Importantly, Koenderink (1984) has shown that convex and concave regions of a contour arise respectively from the projection of convex and saddle regions of a smooth opaque surface.[3] The implication is that, because of how objects self-occlude, the convex-

[3] Convex, concave and saddle points on a surface are defined by their Gaussian curvature, but in this discussion it is sufficient to rely on the intuitive meaning of the words. Anticlastic or hyperbolic points are other words for saddle points, and synclastic or elliptic points are other words for both convex and concave points. If we throw a dart to a smooth surface we would land with probability 1 on one of these three types of locations (convex, concave, saddle). This means that they are fundamental in describing shape, although unfortunately curvature is not all that there is to solid shape (Koenderink, 1990).

ities and concavities of a 2D contour are informative about rims and therefore solid shape. We refer to this information also as contour polarity because if we compute curvature along the contour, the values for convex and concave regions have opposite sign.

A second important fact is that when arbitrary solid shapes meet and interpenetrate, they form concave creases. If one allows for some smoothing, the points along the crease are saddle points that can be seen in the image (i.e. 2D projection) as concavities (Hoffman & Richards, 1984). A crease is a concave discontinuity, but in the more general case of a smooth surface, this is a point where the curvature reaches a minimum. Hoffman and Richards (1984) consequently argue that the visual system should see shapes as composed of parts separated by minima of curvature. This part decomposition can be useful for the purpose of recognizing a complex shape on the basis of its structural description (Biederman, 1987; Marr & Nishihara, 1978).

In summary, although there is no a priori reason for the visual system to necessarily encode contour curvature information, there are good reasons rooted in geometry to expect that any system interested in solid shape should be attuned to such information.

Bertamini (2001) (starting from an effect noted by Gibson, 1994) has recently shown that observers are better at judging the position of a vertex if it is perceived as convex. That is, information about the position of convex vertices is more readily available than information about the position of concave vertices. This finding demonstrates the importance of the polarity of contour curvature, i.e. convexities and concavities. Bertamini argued that if objects are parsed into parts separated by concavities (minima of curvature; Hoffman & Richards, 1984), convex regions are represented as parts and have positions, whereas concave regions are boundaries between parts (see also Attneave, 1954). This is a novel approach to the issue of structural information and perceived parts. Computational considerations have long been used to argue for the existence of shape and volume primitives for object recognition (e.g. Biederman, 1987; Marr & Nishihara, 1978). However, in this paper and in Bertamini (2001) the task is not one of object recognition, yet parts play an important role because they are the by-product of how we represent solid shape, and by definition they have the property of having a position. To reiterate this concept, for us a part is by definition a shape with a position and (probably) an orientation.

It may be interesting to look at convexities and concavities from an historical perspective as well. Ibn al-Haytham (known in the West as Alhazen) was writing about the importance of this type of information for perception of shape about 1000 years ago (ca. 1030) in his book on Optics (as cited in Norman, Phillips, & Ross, 2001). Within the Gestalt school the manipulation of this variable was started very early by Bahnsen (1928), a student of Rubin, and especially by Kanizsa and Gerbino (1976), who confirmed that convexity contributed to prägnanz or good form. Much more recently, a paper by Norman et al. (2001) probably deserves mentioning for the ecological validity of their stimuli. They used silhouettes of potatoes (*Ipomoea batatas*) and asked people to reproduce their shapes on paper using only a limited number of points. People systematically chose the maxima of curvature.

For a more in depth discussion of the importance of curvature polarity for perceiving solid shape see Hoffman and Richards (1984), Hoffman (1998), and Koenderink (1984, 1990). For empirical evidence about the fast and obligatory encoding of curvature polarity see Baylis and Cale (2001), Baylis and Driver (1995, 2001a), Bertamini (2001), Bertamini, Friedenberg, and Argyle (2002), Hoffman and Singh (1997), Norman et al. (2001) and Singh, Seyranian, and Hoffman (1999).

1.4. A solution to the paradox

To try and draw together all of these issues, let us consider again Fig. 1a. One difficulty in remembering a background region is due to the different shape that the region would have had if it had been seen as a figure. Had it been a figure, curvature polarity would have been completely different. Another factor in Fig. 1a is that the uniform regions of background have boundaries that belong to more than one separate object. By avoiding this problem, perhaps a clearer example is given in Fig. 1b, based on an observation by Attneave (1974), as discussed in Hoffman (1998). The difference between two halves of a disk is perceptually strong, because convex regions on one side correspond to concave regions on the other. Note how hard it is to recognize that in A and B the two boundaries are in fact congruent, simply because the inside of the region has changed and with it the curvature polarity. On the other hand, A and C are not congruent (they match under a reflection only) and yet they look rather similar (because the curvature polarity is the same). Another way to describe this phenomenon is that it is difficult to judge the shape of contours (the line that divides the circle) and ignore the fact that those contours are the margins of different surfaces.

In summary, although a figure/ground reversal does not change the contours per se, it does change the information encoded. The inside becomes outside and curvature polarity is reversed. When the memory for a hole is tested using the same configuration (the same hole), there is no difficulty in remembering it (Palmer, 1999), not because the hole is represented as having its own shape, but because the shape of the overall figure (with a hole in it) has not changed. It is only when memory for a hole (ground) is tested using an object (figure) with an identical outline that the task becomes difficult, because their shapes are different in terms of curvature polarity (Fig. 1b).

In this paper we have exploited the convexity advantage discussed above (Bertamini, 2001) using a task in which observers had to judge the relative position of two vertices (task originally introduced by Baylis & Driver, 1993). We tested whether regions that are perceived as holes are processed in exactly the *opposite* way from regions that are perceived as objects.

Consider a simple circle seen as a figure; this region has a strictly convex contour (positive curvature). The same circle seen as a hole, by definition, has a strictly concave contour (negative curvature). Consider now the six-sided objects of Fig. 2. The task is to judge which of the two vertices on the side is lower (left or right). The comparison between the Barrel (convex vertices) and the Hourglass (concave vertices) shows that people are better in the case of the Barrel, even though the vertices are farther apart (because the areas of the two shapes is kept the same) (Gibson, 1994). To discover whether this is a consequence of a change in polarity of the contours we adopted a design that included Barrel- and Hourglass-shaped holes. We show that when simple shapes are depicted as holes there is a reversal of the pattern seen for objects with identical contours. It is easier to judge the position of the vertices for the Barrel-shaped object than the Barrel-shaped hole. Conversely, it is easier to judge the position of the vertices for the Hourglass-shaped hole than the

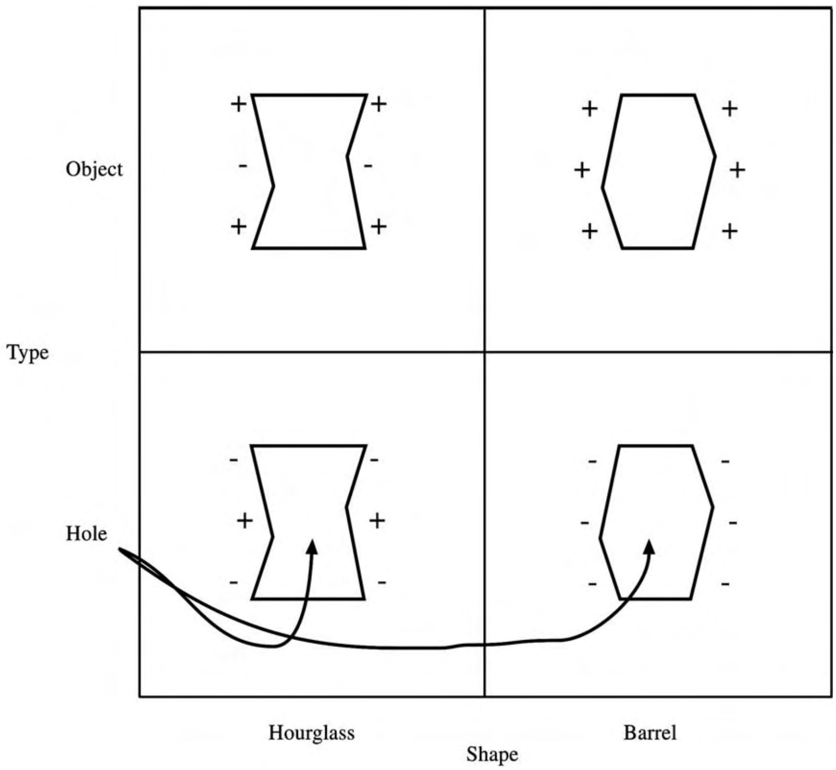

Fig. 2. Two shapes were used in all experiments. Both shapes were irregular hexagons; for mnemonic purposes we label the shape on the right Barrel and the one on the left Hourglass. In this figure we also label with + and − the convex and concave vertices, and in the bottom row we show how such information changes when the shapes are perceived as holes. Note in particular the change from convex to concave and vice versa of the two vertices at the side of the figures, as these are the only vertices carrying the information necessary to perform the task (i.e. one of these vertices was always vertically higher or lower than the other).

Hourglass-shaped object. This is a remarkable change that can only be explained on the basis of polarity inversion, since the geometry of the contours themselves remains unchanged.

Our conclusion is that the shape of a hole is known indirectly via the shape of the object enclosing it. Therefore, our representation of a hole is qualitatively different to our representation of a figure possessing the same contours, and this is due to the drastic differences in part structure between the two. Let us take the example of a fish-shaped hole (discussed in Casati & Varzi, 1994). It is true that people can recognize this as a fish, but our claim is that to do that people must be doing one of two things; either they enforce a figure/ground reversal so that they do not see this region as a hole, or, alternatively, they rely on a slower process that compensates by means of our vast cognitive resources for the fact that perceptually that figure is not the figure of a fish (albeit it is a figure containing enough information to infer the presence of a fish). We are aware that this is a strong claim. Put it in other words, the (perceived) shape of holes is different from the (perceived) shape of objects with the same (congruent) shape. The experiments presented in this paper support this claim. However, in this paper we do not tackle the role of familiarity, and therefore we use simple geometrical shapes.

Research on perception of holes in the literature is limited, with a few exceptions (Cavedon, 1980; Nelson & Palmer, 2001). In generating our stimuli we relied on Bozzi's observations (Bozzi, 1975). He has pointed out that for a hole to be perceived as such the figure that contains it should be perceived as a figure and should therefore have a visible outer boundary, in addition to the 'inner boundary' that forms the edges of the hole; there should be evidence that the background seen through the hole is the same as the background outside the figure; finally, the boundary of the hole should be related, or at least not completely unrelated, to the outer boundary of the figure (for a summary in English of Bozzi's observations see Palmer, 1999, p. 287). We therefore exploited these rules to create stimuli on a computer, reproduced in Fig. 3. The figure containing the hole was drawn in a solid green colour, and the background visible through the hole and outside the

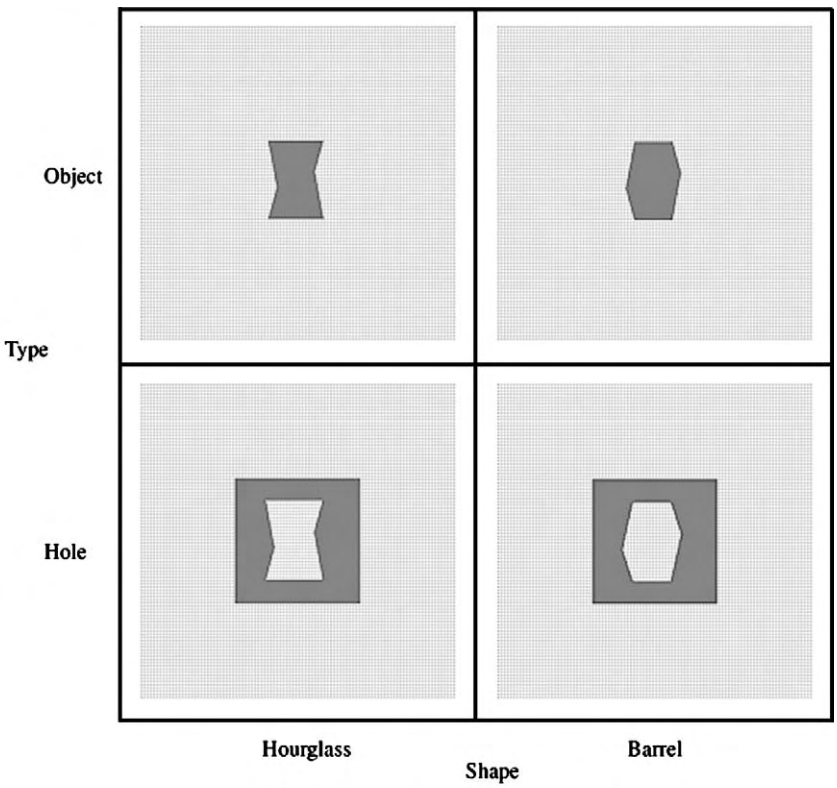

Fig. 3. Examples of stimuli used in Experiment 1. Note that the contours of the shapes themselves are unchanged in the Object and Hole conditions. The background had a texture composed of tiny red diamonds and was present on the screen throughout the experiment. The actual angle formed by the vertices was varied from trial to trial.

figure had a texture to promote the impression of a surface (Experiment 1). Moreover, the textured background was always present on the screen, i.e. was not erased after every trial. Finally, the hole was (roughly) centred on the figure.

Because it is easier to access positional information for convex vertices (Bertamini, 2001) we predicted and found a cross-over interaction between Shape (Barrel and Hourglass) and Type (Object and Hole).

2. General method

Participants sat in a quiet room under conditions of normal illumination. The stimuli were generated and presented by an Apple G4 computer connected to a 17 inch monitor (1250×980 pixels at 75 Hz). The computer also recorded whether each response was correct and the reaction time from stimulus onset in milliseconds using the VideoToolbox functions (Pelli, 1997). Each participant saw 24 practice trials, followed by five blocks of 160 trials each. They were encouraged to take breaks if necessary at the end of each block. All conditions were interleaved and the order of presentation of the trials was randomized for each participant.

The monitor was surrounded by a reduction screen to encourage perception of depth in the display, leaving a circular area of screen with a diameter of 22 degrees of visual angle visible. The average viewing distance was 57 cm. Fig. 3 shows the stimuli for Experiment 1, but the layout was similar in all experiments. A square area of background with a 10 degree side was always present on the screen. In Experiment 1 this area had a texture consisting of small dark red diamonds on a white background. Two shapes were compared, both irregular hexagons, but for illustrative purposes we used the term Barrel for the strictly convex shape, and the term Hourglass for the shape with concavities. In the Object condition, both shapes were 2.5 degrees of visual angle tall. In the Hole condition, the same shapes were holes in a larger square (4 degree side). On average over all the trials, the Barrel and Hourglass shapes had the same area. These shapes were chosen for two reasons. Firstly, they are among the simplest possible shapes in which there are two facing vertices, and secondly, they are similar to those used in previous literature (Baylis & Driver, 1993; Gibson, 1994). The solid colour used for the object was always green. Note that in the Hole condition the same regions, instead of being green objects, were presented as areas of background colour in a green square. However, the position was only approximately centred, because all objects and holes were presented at random small distances from the centre of the screen to (a) minimize visual aftereffects and (b) avoid a strategy of focusing always on one screen location.

The angles of the target vertices were randomly varied between 148 and 168 degrees (acute angle measures). This meant that the steepness of the sides of the shapes varied, as did the distance between the two vertices. This aspect of the stimuli is new with respect to similar displays used in the literature (e.g. Baylis & Driver, 1993) and was introduced to avoid possible learning effects that could take place when a given angle is presented in hundreds of trials.

The vertical offset between the right and left vertices was always fixed to 0.5 degrees of visual angle. Because the task was about which vertex was lower, this vertical offset was the only factor that determined whether 'left' or 'right' was the correct answer. The observers were instructed to look for this offset, and they were told to press the "z" key if the left vertex was lower and the "/" key if the right vertex was lower. The stimuli were verbally described as objects and objects with holes, but there was no pressure in the instructions to organize the stimuli in depth. At the end of the 24 practice trials observers were asked whether they had any questions or would like more practice, in which case a second practice session was started. Both accuracy and speed were stressed, and a brief sound immediately informed the participants of an incorrect response.

3. Experiment 1: the shape of objects and holes

We compared reaction times and errors for the stimuli shown in Fig. 3. We predicted a significant interaction, and more specifically we predicted the interaction to have the following critical aspect. Observers would be faster for the Barrel shape when it was depicted as an Object than when it was a Hole, because in the Object condition the vertices were convex and in the Hole condition they were concave (for evidence of a convexity advantage see Bertamini, 2001). The opposite should occur for the Hourglass shape, because when it was an Object the vertices were concave and when it was a Hole they were convex. Referring to Fig. 3, this means an opposite effect of Type for the two levels of Shape, and therefore the important comparison is within each column of the figure. A less critical aspect is the comparison within the rows of the figure. Responses to the Barrel should be faster than responses to the Hourglass in the Object condition, but the opposite should be true for the Hole condition. However, this second aspect of the interaction relies on Barrels and Hourglasses being roughly the same in difficulty, and this may not be the case. In other words there may be a main effect of Shape, with one shape faster than the other overall. For example, it has been suggested that the Hourglass has a more complex part structure because it is 'bisected' by two concave vertices (Gibson, 1994).

In summary, we predicted an interaction between Shape and Type, and a difference in the direction in which Type affected the two Shapes, with the change from Objects to Holes making the task harder for the Barrel but easier for the Hourglass, due to the polarity reversal.

In Experiment 1 we varied the Type factor according to the figural considerations discussed above (Bozzi, 1975), but in addition we manipulated whether the observers saw the stimuli monocularly or binocularly. This factor may affect depth stratification, as binocular viewing provides clues to the flatness of the display, and therefore may influence whether the objects with holes are seen in front of the background.

3.1. Method

Twenty-eight psychology undergraduates from the University of Liverpool participated in this experiment in return for course credit. Their average age was about 19 years and 25 were female. They all had normal or corrected vision. Fourteen participants did the task binocularly and 14 participants did the task monocularly.

3.2. Results and discussion

Results are shown in Fig. 4. The graphs show the mean reaction times for the four combinations of factors (Barrel–Object, Barrel–Hole, Hourglass–Object, and Hourglass–Hole) averaged across all participants. The error bars are within-subjects standard errors computed according to Loftus and Masson (1994). The lower graphs show the mean percentage of errors for the same four conditions. For this and all subsequent analyses, responses above 3000 ms were excluded as outliers, and a logarithm transformation was used to normalize the distribution (Ulrich & Miller, 1993). For the reaction times analyses incorrect responses were also excluded.

In Fig. 4 we do not collapse the results from the monocular and binocular viewing conditions, but there was no difference between the two conditions according to the analyses of variance (ANOVAs) on reaction time and accuracy reported below.

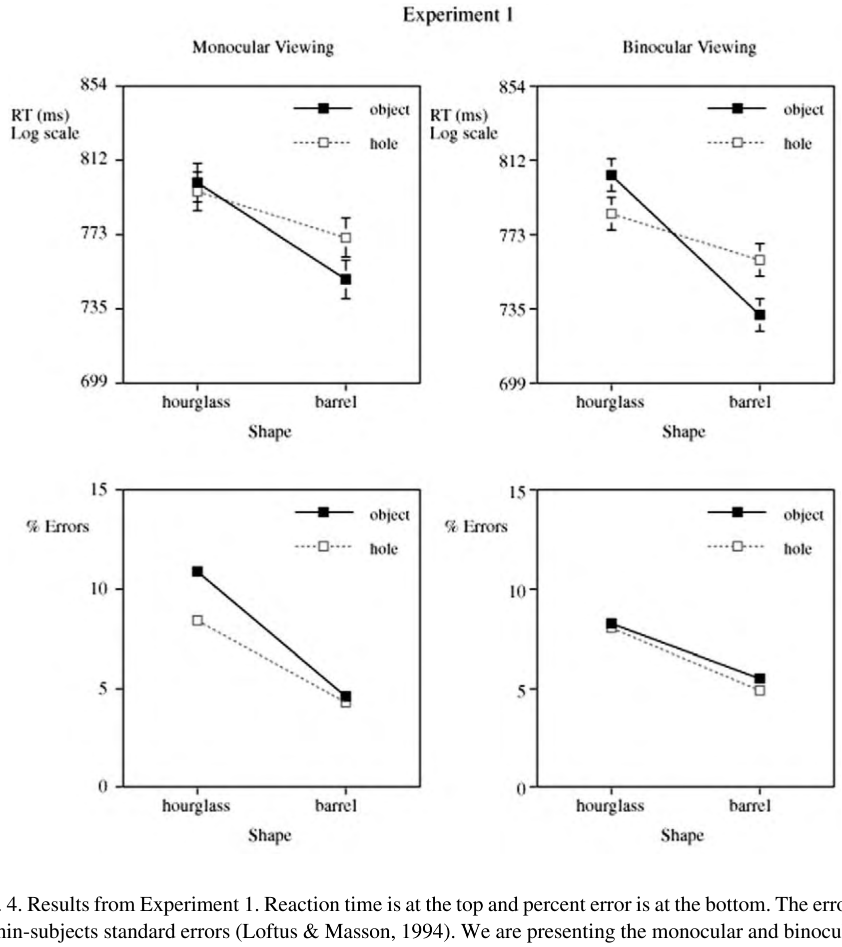

Fig. 4. Results from Experiment 1. Reaction time is at the top and percent error is at the bottom. The error bars are within-subjects standard errors (Loftus & Masson, 1994). We are presenting the monocular and binocular results separately for information, but the difference was not significant.

3.2.1. Reaction times

A three-way mixed ANOVA on Shape (Barrel or Hourglass), Type (Object or Hole) and Viewing (binocular or monocular) showed that Shape was a significant factor ($F(1,26) = 18.66$, $P < 0.001$), with participants faster on the Barrel. Type was also significant ($F(1,26) = 4.92$, $P = 0.035$), with people being slightly quicker for the Object stimuli than the Hole stimuli. The interaction between Shape and Type was highly significant ($F(1,26) = 25.19$, $P < 0.001$). A Scheffé post-hoc test showed that people were significantly faster on the Barrel when it was an Object than when it was a Hole ($P < 0.001$), but they were marginally slower on the Hourglass when it was an Object than when it was a Hole ($P = 0.057$). Finally, there was no effect of Viewing ($F(1,26) = 0.18$, $P = 0.673$) and it did not interact with any of the other factors.

3.2.2. Accuracy

Fig. 4 also shows that accuracy was high. However, Shape was significant ($F(1,26) = 25.78$, $P < 0.001$), with people being more accurate for the Barrel shape. There was a marginal effect of Type ($F(1,26) = 3.92$, $P = 0.058$) and no interaction ($F(1,26) = 1.06$, $P = 0.312$). Finally, Viewing condition had no effect on accuracy ($F(1,26) = 0.22$, $P = 0.641$). An inspection of the graphs shows no sign of a speed–accuracy trade-off.

Experiment 1 confirmed that people were slower on the Hourglass than on the Barrel, across both Object and Hole conditions. People were also faster for Object than Hole stimuli, perhaps because of the different size of the green stimulus (larger in the Hole condition). Most interestingly for us, there was a cross-over interaction between the two factors: people were faster on the Barrel shape when it was presented as an Object than when it was presented as a Hole, and vice versa for the Hourglass (albeit marginally). Ideally we would have liked the reversal (cross-over interaction) to be as clear for the Barrel and for the Hourglass. Possibly this problem is related to the fact that the Hourglass was a harder condition overall. This factor was manipulated in a subsequent experiment (Experiment 3).

We suggest that this interaction is because the polarity of the vertices was reversed, i.e. for the Barrel the vertices were represented as convex parts in the Object trials but as concavities in the Hole trials. The vertices of the Hourglass were more difficult to judge in the Object trials, perhaps because concavities are represented as boundaries between parts, which are not perceived as having a position of their own (Bertamini, 2001). This suggests that the important factor in judging within-shape vertices is not the overall shape but whether those vertices are perceived as being concave or convex, and that this can be manipulated by changing figure and ground so that the shape is perceived as a hole.

Initially we were concerned that having participants view the displays binocularly would reduce their perception of depth in the displays due to binocular disparity information, leading to the hole regions being perceived not as holes but as flat shapes superimposed on rectangles. However, the data obtained with binocular viewing were remarkably consistent with those from the monocular viewing condition. Perhaps this should have been expected all along, because a significant difference in depth need not be necessary for the surface lying on the top to have a hole in it. The similarity between the two viewing conditions suggests that our manipulation of the display did in fact have the effect predicted by Bozzi (1975) in creating the impression of a hole on the basis of figural factors.

Our findings are consistent with those recently reported by Tsal, Lamy, and Ilan (2000), who used conditions similar to our Barrel–Object and Barrel–Hole. However, they were not investigating perception of contour polarity. Their interpretation of the poorer performance in the hole condition ("enclosure" in their terminology) is that it requires attention to spread over a larger area. This is possible, and is a factor that may have been confounded with the effect of number of objects in the past (Baylis & Driver, 1993). However, in this paper we remain neutral with respect to the debate about object-based versus space-based attention. By comparing the Barrel and Hourglass stimuli we have found an interaction which cannot be explained on the basis of either the number of objects (one in both the Object and Hole conditions) or the spatial extent of the area attended (the same in the Barrel and Hourglass conditions).

4. Experiment 2: solid colours

Although we used a textured background to encourage depth stratification for the Hole conditions, we wondered whether this might have affected the results in some unpredicted way. We therefore ran a control experiment using dark red shapes on a uniform green background, to see whether the interaction of Shape and Type would still occur under these conditions.

4.1. Method

Fourteen psychology undergraduates from the University of Liverpool participated in return for course credit. None had participated in Experiment 1. All participants did the task binocularly. The equipment and procedure were identical to those in Experiment 1 except that the texture was replaced by a solid green colour and the green figure was replaced by a solid red colour (see Fig. 5). The green was the same colour used for the figure in Experiment 1, and the red was the same colour used for the squares of the texture in Experiment 1. The background was darker than the old texture because it was now a solid colour. If our conclusions from Experiment 1 were correct we should again find a significant interaction between Shape and Type, notwithstanding the changes in colours.

4.2. Results and discussion

The results for both reaction times and errors can be seen in Fig. 6. We ran a series of ANOVAs using the same design as in Experiment 1.

4.2.1. Reaction times

Once again, Shape was a significant factor ($F(3, 13) = 24.35$, $P < 0.001$), with participants faster on the Barrel. There was no main effect of Type ($F(1, 13) = 0.85$, $P = 0.374$), but a significant interaction ($F(3, 13) = 5.67$, $P = 0.033$).

4.2.2. Accuracy

Accuracy was even higher than in the first experiment. However, there was a significant effect of Shape ($F(3, 13) = 21.07$, $P < 0.001$) but not of Type ($F(1, 13) = 1.89$,

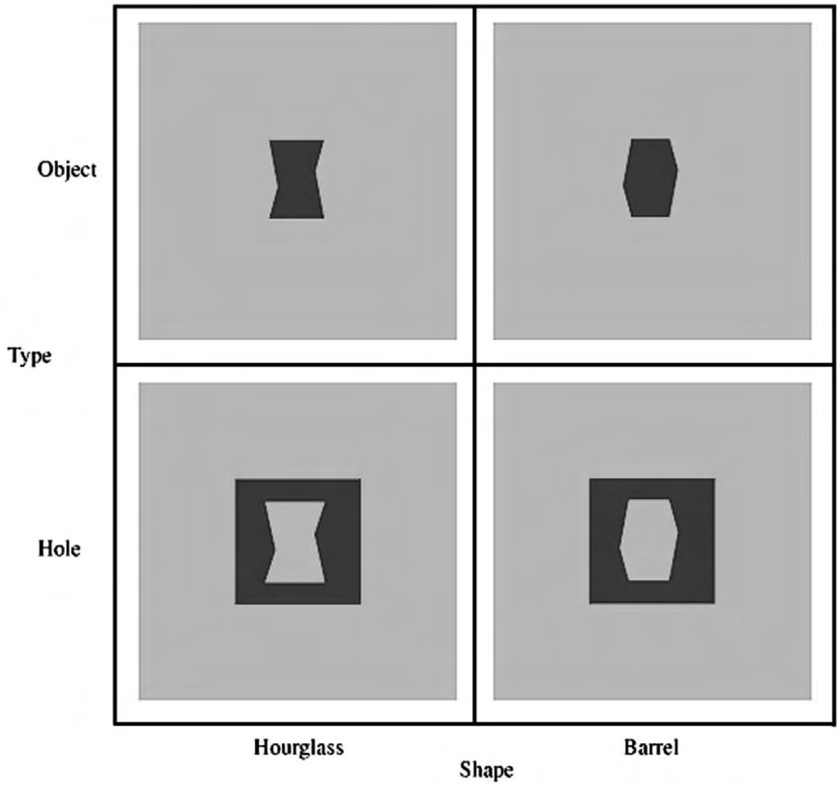

Fig. 5. Examples of stimuli used in Experiment 2. The only difference with respect to the stimuli in Experiment 1 was the elimination of the texture. The figure was always red and the background always green.

$P = 0.193$), and no interaction ($F(3, 13) = 2.70$, $P = 0.124$). An inspection of the graphs shows no sign of a speed–accuracy trade-off.

This experiment replicated the findings of Experiment 1, although it is interesting that the participants in this condition were faster than those in Experiment 1. The means differ from those of Experiment 1 by between 80 ms (for the Barrel–Object condition) and 150 ms (for the two Hourglass conditions). This may be because removing the texture decreased visual complexity and made the task simpler. However, this control experiment confirms that it is easier to compare two vertices within a shape if they are convex than if they are concave.

5. Experiment 3: adjustment of difficulty

Experiments 1 and 2 confirmed an interaction between Shape and Type, even when overall the task was easier. Nevertheless, in both experiments the effect of Shape was significant, showing that the Hourglass was in general harder than the Barrel. Because

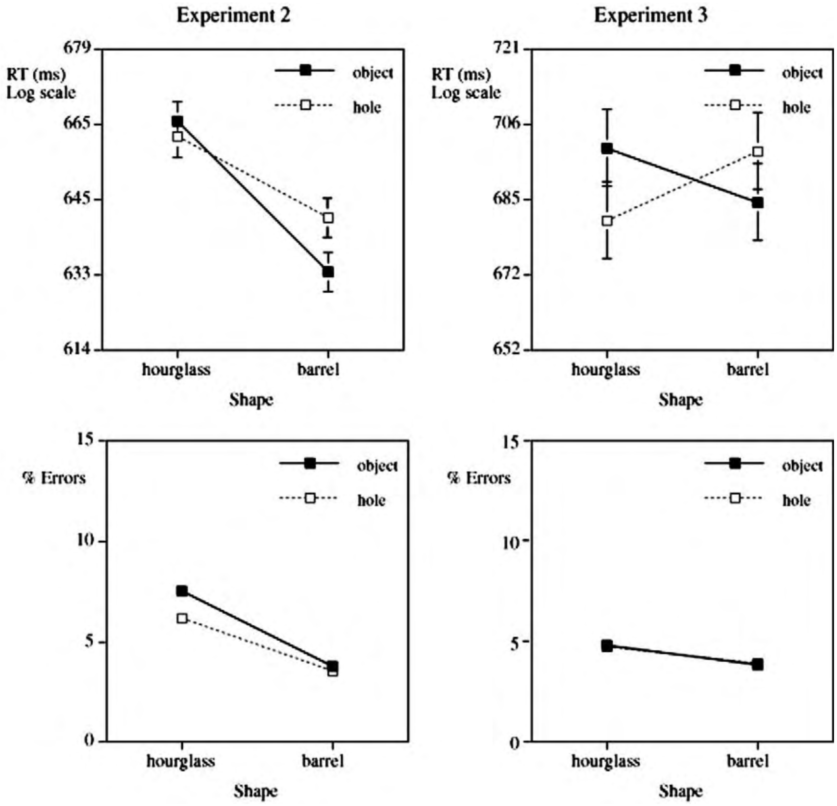

Fig. 6. Results from Experiment 2 (solid colours) and 3 (adjusted difficulty). Reaction time is at the top and percent error is at the bottom. The error bars are within-subjects standard errors.

comparing differences in performance at different points on a range of difficulty is problematic we wanted to ensure that the interaction was present when all conditions were in the same range. Therefore, Experiment 3 tested the hypothesis that the interaction would still be present if the relative difficulties of the two shapes were to be adjusted so as to require comparable reaction time. A change from Object to Hole should make the Barrel task harder and the Hourglass task easier, as before, but in this new scenario we also expect that a change from Barrel to Hourglass should make the Object task harder and the Hole task easier (an effect that in Experiments 1 and 2 was hidden by an overall difference in difficulty between the two shapes). As can be seen in Fig. 6, this type of cross-over interaction was indeed confirmed.

5.1. Method

Sixteen observers participated in return for course credit or a small monetary reward. The procedure was identical to Experiment 1 except that the process of generating the vertices was adjusted. As before there were two vertices on the sides of the shapes which participants had to look at in order to perform the task. The range of the angles formed by these vertices was reduced to only two values; for the Barrel they were 161 and 156 degrees, whilst for the Hourglass they were 156 and 152 degrees. The smaller angles (sharper vertices) used for the Hourglass should make the task easier in relative terms, and the exact values were chosen after piloting. We predict again a significant interaction between Shape and Type, but this time there should be no effect of Shape.

5.2. Results and discussion

The results for both reaction times and errors can be seen in Fig. 6. We ran an ANOVA using the same steps and design as in Experiments 1 and 2. Shape was not a significant factor ($F(1, 15) = 0.02$, $P = 0.897$), and neither was there an effect of Type ($F(1, 15) = 0.18$, $P = 0.675$). However, the interaction was significant ($F(1, 15) = 13.64$, $P = 0.002$). Post-hoc Scheffé tests on the means confirmed that for the Barrel, the Object condition was faster than the Hole condition ($P = 0.048$), whilst for the Hourglass the Object condition was significantly slower than the Hole condition ($P = 0.008$). Accuracy was again extremely high, and as a result no factor was significant. An inspection of the graphs in Fig. 6 shows no sign of a speed–accuracy trade-off.

This experiment replicated the findings of Experiments 1 and 2. The important contribution of Experiment 3 is that it proves that eliminating a difference in difficulty between the two shapes does not eliminate the cross-over interaction. On the contrary, it supports our original hypothesis that because convex vertices define the structural parts of an object, their position is always more readily available than the position of concave vertices.

6. General discussion

Although holes are very interesting entities for study in themselves, they become even more interesting with respect to the study of how shape information is encoded and represented. In this paper we found evidence to support the central role of curvature polarity in shape perception. By changing the nature of a given closed contour from an object to a hole we can achieve a reversal of curvature polarity without any change in the actual contours (i.e. edge information). This reversal is a special case of a figure/ground reversal, which has already been shown to affect performance in visual search (Humphreys & Müller, 2000), contour matching (Driver & Baylis, 1996) and judgements about position (Bertamini, 2001). Unlike other figure/ground reversals, holes are unique in that there is no change in the actual (and presumably the perceived) closure of the region. In fact, the figure/ground reversal in the present experiments was achieved purely on the basis of contextual information (Bozzi, 1975). Using holes, a curvature polarity reversal can be achieved without altering factors such as closure and number of objects.

The question arises of how a change of curvature polarity takes place. Is this inversion a slow and serial process, and does it require attention? The Barrel and the Hourglass have six vertices, but for the Barrel they are all convex, whilst for the Hourglass four are convex and two are concave (Fig. 2). We mentioned that it has been suggested that this makes the

Hourglass more complex and responses to it slower (Gibson, 1994). In our argument we have claimed that curvature polarity is a very basic aspect of shape, and it is extracted fast and obligatorily. However, the precise role of attention in curvature extraction is beyond the scope of the present paper. As for the complexity of the Hourglass shape, comparing the figures-with-holes reveals that the figure with Hourglass-shaped hole would still be more complex than the figure with Barrel-shaped hole, and therefore this difference cannot explain the interaction.

It is important to note that in all the experiments presented here, observers could have responded purely on the basis of the shape of contours disregarding which side of the contour was figure and which was ground. There was nothing in the task or in the instructions that forced observers to pay attention to figure/ground organization. But had they treated the contours as edges defined only by colour or luminance, no interaction between Shape and Type would have been found. The interaction was present because figure/ground organization is obligatory and curvature polarity information cannot be ignored. This is consistent with what was found by Driver and Baylis (1996) and Bertamini et al. (2002).

What we call figures or objects in our experiments could be called *surfaces*, and in this sense our findings are also consistent with the evidence of a primary role of surfaces for the human visual system (e.g. Lappin & Craft, 2000; Nakayama, He, & Shimojo, 1995; Nakayama & Shimojo, 1992). Surfaces may be defined by contours and lines in the geometry we learn at school but this does not imply that they are secondary or that they are constructed from lines by the human visual system. If visual organisms have evolved to extract relevant visual information to survive in a world of solid shapes, then convex and concave regions of contours are directly informative about solid shape. This difference helps us to understand the phenomena described in Fig. 1a,b. Moreover, Bertamini (2001) suggested that convex regions define parts of solid objects that need a description in a way that is qualitatively different from the description of a concave region, namely positional information should be more directly available for convex parts. If this is true we can predict differences between two congruent outlines when they are perceived as figure and hole. In this paper we empirically confirmed these predictions.

Anecdotal support for this view that convexity is critical for how we represent shape can be found also in the following quote: "In general the surfaces of things growing or blown up from the inside tend to possess positive Gaussian curvature. […] the sculpture of many cultures reveals this very clearly. Very often the negatively curved parts are reduced to narrow V-shaped grooves between the bulging ovoid parts. An extreme example is the famous "Venus of Willendorf" from paleolithic origin (ca. 11000 B.C.). Clearly this fact that we all seem to know from introspection is not a short-lived "cultural whim", but somehow pervades human visual perception throughout known history and over all cultures." (Koenderink, 1990, p. 251).

Note that simple contours and surfaces were used in our experiments, but the logic of the discussion relies on the fact that even such pictorial stimuli must be understood in the context of perceiving and representing solid shapes. Contours are informative about rims of self-occluding surfaces in space. In other words, the experimenter's choice of simple stimuli cannot turn off the sophisticated visual intelligence of a human observer, and the same machinery will be engaged by any visual task, although simple stimuli can reveal important underlying principles.

The importance of describing the information available to the observer in terms of surfaces and media (an idea first introduced by Gibson, 1979) can be further clarified by contrasting it with possible alternatives. As discussed in Section 1, much of the problems with holes stem from the fact that they are not objects whereas at the same time they are uniform connected regions (a concept introduced by Palmer & Rock, 1994). According to Palmer and Rock (1994) uniform connected regions are the initial units of perceptual organization. Both material objects and uniform connected regions have intuitive appeal as the basic units of perception. But a visual system interested in material objects is construed as a Physicist, and a visual system interested in uniform connected regions is construed as a serial Computer that has to start from a 2D image similar to a mosaic to recover 3D information (for a critique see, for example, Bruno, Bertamini, & Domini, 1997). As discussed earlier, we prefer to think of the visual system as a Geometer, interested first and foremost in solid shape.

It could be argued that in all our experiments there was some ambiguity in the display. This is true, however we have conducted a separate set of experiments in which the display was not ambiguous at all, because of binocular disparity. By using random dot stereograms we have presented surfaces at different depths with or without holes. The shapes are only visible after fusing the two stereograms so we can be confident that a hole in a surface is perceived as such. Consistent with this more powerful way in which a hole was created, we have found even stronger interactions than those reported in this paper (Bertamini & Mosca, 2002).

Many of the issues mentioned in this discussion can be followed up in future research, and some of this work is under way. Timing issues can be tested by varying presentation time or adopting a priming paradigm (e.g. Baylis & Cale, 2001). Issues to do with attention can be looked at, for example, with secondary tasks. The role of convexity in creating complete objects can be detected indirectly by tasks requiring fine shape judgements (e.g. Liu, Jacobs, & Basri, 1999). The basic difference in the representations of a figure and a hole even when their contours are congruent could be detected with imaging techniques which would show the important cortical regions involved in processing shape information modulated by figure/ground organization (e.g. Baylis & Driver, 2001b; Kourtzi & Kanwisher, 2001). Holes with meaningful shapes could be used to see whether recognition of a hole as a particular object will substantially affect its representation (e.g. Peterson et al., 2000).

There are even some perhaps more difficult philosophical questions that must be addressed. Should perception and ontology agree or can there exist fundamental differences between what holes are (a question of ontology), and what they are for the visual system?

7. Conclusions

The interaction reported in this paper is consistent with the hypothesis that the position of a convex region is more readily available to the observer (Bertamini, 2001; Gibson, 1994).

Moreover, with respect to the paradox of how people can remember the shape of a hole, we conclude that a hole is defined by the contour of the enclosing object, rather than the hole itself possessing the contour. To repeat the argument, consider the case of a rectangle containing an Hourglass-shaped hole (Figs. 2 and 3): the object in this case has two contours – a square contour on the outside and the Hourglass contour which contains two protruding parts pointing inwards towards a central void. Even if this hole is always represented by the shape of the object-with-hole, this would not prevent observers from recognizing the shape of the hole when asked to remember it over time. This interpretation is therefore entirely consistent with the findings of Rock, Palmer, and Hume (cited in Palmer, 1999). In summary, because the shapes of holes and objects have curvature with opposite polarity (by definition) they are perceived as having different shapes. Therefore, we have demonstrated that (a) contour assignment and curvature polarity have a critical and obligatory role in the representation of shape in the human visual system, and that (b) holes are promising material for the empirical study of shape representation. However, none of what we have discovered so far contradicts the tenet of Gestalt theory that says that boundaries belong to the figure and only to the figure (Koffka, 1935; Rubin, 1921).

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